Internet Measurement and Data Analysis (11)

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review of previous class

Class 10 Anomaly detection and machine learning (12/05)

- Anomaly detection
- Machine Learning
- SPAM filtering and Bayes theorem
- exercise: naive Bayesian filter

today's topics

Class 11 Data Mining

- Pattern extraction
- Classification
- Clustering
- exercise: clustering

data mining

- huge volume of data
  - difficult to handle with traditional methods
  - need to extract information hidden in data that is not readily evident
- Data Mining
  - huge volume, multi-dimensional diverse data, non-trivial distributions
  - methods often derived from ideas in machine learning, AI, pattern recognition, statistics, database, signal processing
- data processing becomes practical by growing computing power (e.g., cloud computing)

Data Mining methods

definition: non-trivial extraction of implicit, previously unknown and potentially useful information from data

- pattern extraction: find existing models and patterns in data
  - correlation
  - time-series
- classification: automatically create new classes that do not exist in the original data
  - rule-based methods
  - naive Bayesian filter
  - neural networks
  - support vector machine (SVM)
  - dimensionality reduction (e.g., PCA)
- clustering: compute the distance (or similarity) between data points and group them
  - distance based, density based, graph based
  - k-means, DBSCAN
- anomaly detection: find deviation from normal state using statistical methods
  - univariate, multivariate
  - outlier detection

distances (review)

various distances

- Euclidean distance
- standardized Euclidean distance
- Minkowski distance
- Mahalanobis distance

similarities

- binary vector similarities
- n-dimensional vector similarities

properties of distance

a metric of distance $d(x, y)$ between 2 points (x, y) in space
positivity

$$d(x, y) \geq 0$$

$$d(x, y) = 0 \Leftrightarrow x = y$$

symmetry

$$d(x, y) = d(y, x)$$

triangle inequality

$$d(x, z) \leq d(x, y) + d(y, z)$$

Euclidean distance

word "distance" usually means "Euclidean distance"
a distance of 2 points (x, y) in a n-dimensional space

$$d(x,y) = \sqrt{\sum_{k=1}^{n}(x_k - y_k)^2}$$

standardized Euclidean distance

- when variances are different among variables, distances are affected.
- standard Euclidean distance: normalized by dividing the Euclidean distance by the variance of each variable

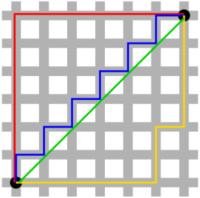
$$d(x, y) = \sqrt{\sum_{k=1}^{n} \frac{(x_k - y_k)^2}{s_k^2}}$$

Minkowski distance

generalization of Euclidean distance: as parameter r grows, a short cut crossing different axes is preferred more

$$d(x, y) = (\sum_{k=1}^{n} |x_k - y_k|^r)^{\frac{1}{r}}$$

- $r = 1$: Manhattan distance
  - Hamming distance: for 2 strings of equal length, the number of positions at which the corresponding symbols are different.
  - example: the hamming distance of 111111 and 101010 is 3
- $r = 2$: Euclidean distance



Manhattan distance vs. Euclidean distance

vector norm (1/2)

vector norm: the length of a vector

$$\|x\| \text{ where } x \text{ is a vector}$$

the l_n-norm of x is defined by Minkowski distance as

$$\|x\|_n = \sqrt[n]{\sum_i |x_i|^n}$$

l_0-norm: the total number of non-zero elements in a vector

$$\|x\|_0 = \#(i|x_i \neq 0)$$

l_1-norm: sum of absolute difference

$$\|x\|_1 = \sum_i |x_i|$$

l_2-norm: Euclidean distance

$$\|x\|_2 = \sqrt{\sum_i |x_i|^2}$$

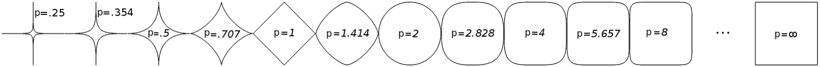
l_∞-norm: the maximum entry's magnitude of a vector

$$\|x\|_\infty = max(|x_i|)$$

vector norm (2/2)

For the example vector $x = (1, 2, 3)$

$$
\begin{array}{rc}
\|x\|_0 & 3 = 3.000 \\
\|x\|_1 & 6 = 6.000 \\
\|x\|_2 & \sqrt{14} = 3.742 \\
\|x\|_3 & 6^{2/3} = 3.302 \\
\|x\|_4 & 2^{1/4}\sqrt{7} = 3.146 \\
\|x\|_\infty & 3 = 3.000
\end{array}
$$



unit circles of l_p-norm with various values of p

Mahalanobis distance

a distance that takes correlations into account, when correlation exists between variables

$$mahalanobis(x, y) = (x - y)\Sigma^{-1}(x - y)^T$$

here, Σ^{-1} is the inverse matrix of its covariance matrix

similarities

similarity

- numerical measure of how alike 2 data objects are

properties of similarity

positivity

$$0 \leq s(x, y) \leq 1$$

$$s(x, y) = 1 \Leftrightarrow x = y$$

symmetry

$$s(x, y) = s(y, x)$$

in general, triangle inequality does not apply to similarities

similarity between binary vectors

Jaccard coefficient

- used for similarity between binary vectors in which the occurrences of 1 is much smaller than the occurrences of 0
- example: as a metric of similarity by occurrences of words in documents
- many words do not appear in both documents $\Rightarrow$ not considered
- the following table shows the relationship of each item

		vector y	
		1	0
vector x	1	n_{11}	n_{10}
	0	n_{01}	n_{00}

Jaccard coefficient:

$$J = \frac{n_{11}}{n_{11} + n_{10} + n_{01}}$$

similarity between vectors

similarity between (non-binary) vectors

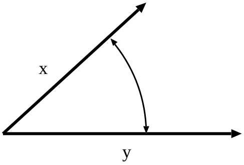
- example: similarity of documents where frequencies of words are also taken into consideration

cosine similarity

- take the angle (cosine) of (x, y) of vectors
- normalized by the length of the vector $\Rightarrow$ length is not considered

$$cos(x, y) = \frac{x \cdot y}{\|x\| \|y\|}$$

$x \cdot y = \sum_{k=1}^{n} x_k y_k$: product of vectors
$\|x\| = \sqrt{\sum_{k=1}^{n} x_k^2} = \sqrt{x \cdot x}$: length of the vector



example: cosine similarity

$$x = 3 \;\; 2 \;\; 0 \;\; 5 \;\; 0 \;\; 0 \;\; 0 \;\; 2 \;\; 0 \;\; 0$$
$$y = 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 2$$

$$x \cdot y = 3 * 1 + 2 * 1 = 5$$
$$\|x\| = \sqrt{3 * 3 + 2 * 2 + 5 * 5 + 2 * 2} = \sqrt{42} = 6.481$$
$$\|y\| = \sqrt{1 * 1 + 1 * 1 + 2 * 2} = \sqrt{6} = 2.449$$

$$cos(x, y) = \frac{5}{6.481 * 2.449} = 0.315$$

clustering

important technique for classifying data with complex relationship

compute the distance (or similarity) of variables to make them into groups

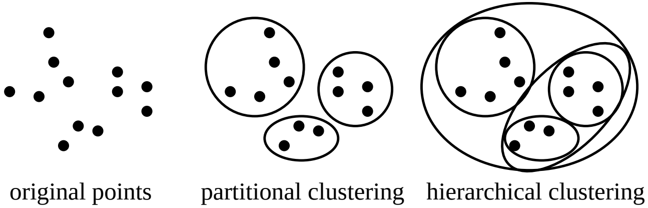
- to classify and understand data
- to summarize data

various applications

- business: grouping customers for marketing purposes
- meteorology: finding patterns in complex weather data
- biology: classifying genes and proteins
- medical science and pharmacy: complex relationship of symptoms and effects

clustering methods

- partitional clustering
  - k-means method
- hierarchical clustering
  - MST method
  - DBSCAN method



k-means method

- partitional clustering
- specify the number of cluster, k
- basic algorithm is simple
  - each cluster has centroid (usually mean)
  - assign each object to the closest cluster
  - repeat re-computation of centroids and cluster assignments
- limitations
  - need to specify the number of clusters, k, beforehand
  - sensitive to the selection of initial points
  - clusters are supposed to have similar sizes and densities, and a round shape
  - sensitive to outliers

basic k-means algorithm:

```
1: select k points randomly as the initial centroids
2: repeat
3:        form k clusters by assigning all points to the closest centroid
4:        recompute the centroid of each cluster
5: until the centroids don't change
```

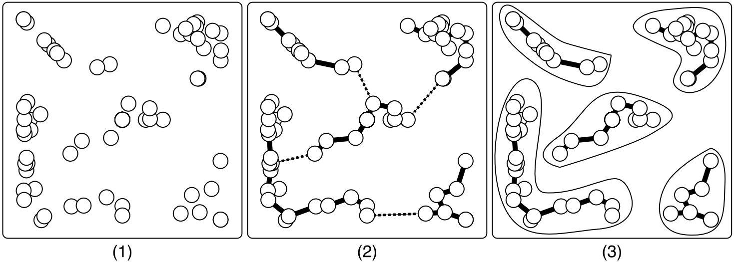
hierarchical clustering

- generate clusters using a tree structure
  - the cluster structure can be explained by the tree
- no need to specify the number of clusters beforehand
- 2 approaches
  - agglomerative: start with data points as individual clusters, and repeat merging the closest clusters
  - divisive: start with one all-inclusive cluster, and repeat splitting clusters

MST clustering

Minimum Spanning Tree clustering

- divisive hierarchical clustering
- start with an arbitrary point, and create MST
- repeat dividing clusters by removing the longest edge



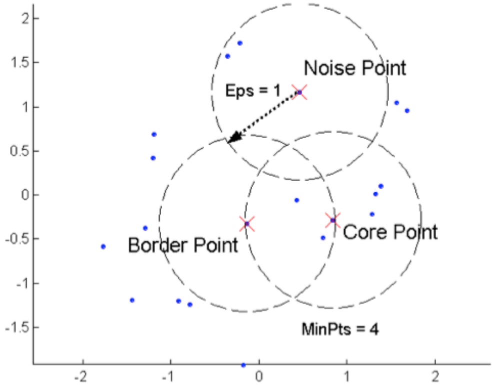
DBSCAN

Density-Based Spatial Clustering

- density-based: combine data points within the specified distance
- can extract arbitrary (non-round) shapes of clusters
- robust against noise and outliers
- distance threshold Eps and point threshold $MinPts$
  - Core points: within the distance Eps, more than $MinPts$ neighbors exist
  - Border points: not Core, but have a core within the distance Eps
  - Noise points: have no core within the distance Eps
- limitations: clusters with different densities, or with large number of parameters

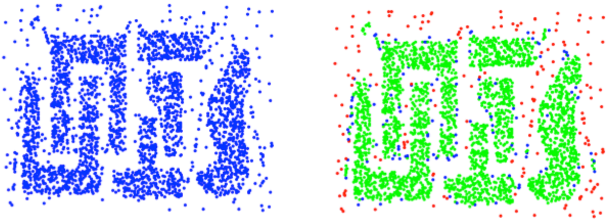
DBSCAN algorithm:
1: label all points as core, border, or noise points
2: eliminate noise points
3: put an edge between all core points that are within Eps of each other
4: make each group of connected core points into a separate cluster
5: assign each border point to one of the clusters of its associated core points

DBSCAN: Core, Border, and Noise Points



source: Tan, Steinbach, Kumer. Introduction to Data Mining

DBSCAN: example of Core, Border, and Noise Points



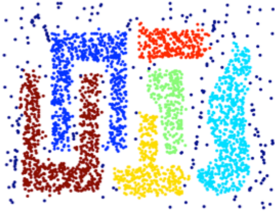
Original Points

Point types: core, border and noise

Eps = 10, MinPts = 4

source: Tan, Steinbach, Kumer. Introduction to Data Mining

DBSCAN: example clusters



Clusters

source: Tan, Steinbach, Kumer. Introduction to Data Mining

previous exercise: SPAM filtering

- SPAM filtering using naive bayesian classifier
  - based on the code from “Programming Collective Intelligence” Chapter 6

```
% ruby naivebayes.rb
classifying "quick rabbit" => good
classifying "quick money" => bad
```

naive bayesian classifier for the exercise

compute the propbability of a document to be classified into a specific category by words appearing in the dicument

$$P(C)\prod_{i=1}^{n} P(x_i|C)$$

- $P(C)$: the probability of the category
- $\prod_{i=1}^{n} P(x_i|C)$: product of the conditional probability of each word in the category

select the category with the highest probability

- threshold： the probability of the best category should be $thresh$ times higher than that of the second best category

SPAM classifier script

- training and classifier

```
# create a classifier instance
cl = NaiveBayes.new

# training
cl.train('Nobody owns the water.','good')
cl.train('the quick rabbit jumps fences','good')
cl.train('buy pharmaceuticals now','bad')
cl.train('make quick money at the online casino','bad')
cl.train('the quick brown fox jumps','good')

# classify
sample_data = [ "quick rabbit", "quick money" ]

sample_data.each do |s|
  print "classifying \"#{s}\" => "
  puts cl.classify(s, default="unknown")
end
```

script: Classifier Class (1/2)

```
# feature extraction
def getwords(doc)
  words = doc.split(/\W+/)
  words.map!{|w| w.downcase}
  words.select{|w| w.length < 20 && w.length > 2 }.uniq
end

# base class for classifier
class Classifier
  def initialize
    # initialize arrays for feature counts, category counts
    @fc, @cc = {}, {}
  end

  def getfeatures(doc)
    getwords(doc)
  end

  # increment feature/category count
  def incf(f, cat)
    @fc[f] ||= {}
    @fc[f][cat] ||= 0
    @fc[f][cat] += 1
  end

  # increment category count
  def incc(cat)
    @cc[cat] ||= 0
    @cc[cat] += 1
  end

  ...
```

script: Classifier Class (2/2)

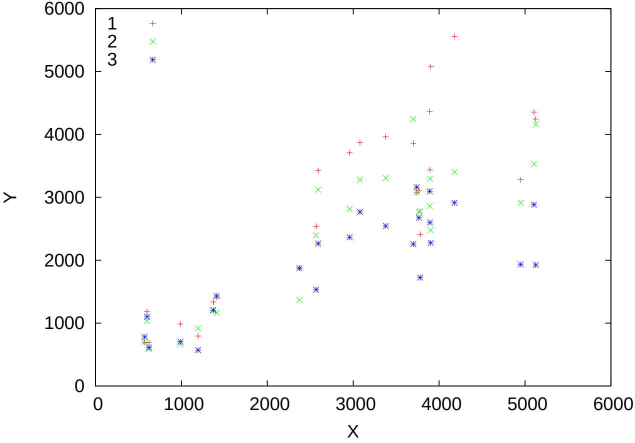
```
  def fprob(f,cat)
    if catcount(cat) == 0
      return 0.0
    end

    # the total number of times this feature appeared in this
    # category divided by the total number of items in this category
    Float(fcount(f, cat)) / catcount(cat)
  end

  def weightedprob(f, cat, weight=1.0, ap=0.5)
    # calculate current probability
    basicprob = fprob(f, cat)
    # count the number of times this feature has appeared in all categories
    totals = 0
    categories.each do |c|
      totals += fcount(f,c)
    end
    # calculate the weighted average
    ((weight * ap) + (totals * basicprob)) / (weight + totals)
  end

  def train(item, cat)
    features = getfeatures(item)
    features.each do |f|
      incf(f, cat)
    end
    incc(cat)
  end
end
```

script: NaiveBayes Class

```
# naive baysian classifier
class NaiveBayes < Classifier
  def initialize
    super
    @thresholds = {}
  end

  def docprob(item, cat)
    features = getfeatures(item)
    # multiply the probabilities of all the features together
    p = 1.0
    features.each do |f|
      p *= weightedprob(f, cat)
    end
    return p
  end

  def prob(item, cat)
    catprob = Float(catcount(cat)) / totalcount
    docprob = docprob(item, cat)
    return docprob * catprob
  end

  def classify(item, default=nil)
    # find the category with the highest probability
    probs, max, best = {}, 0.0, nil
    categories.each do |cat|
      probs[cat] = prob(item, cat)
      if probs[cat] > max
        max = probs[cat]
        best = cat
      end
    end
    # make sure the probability exceeds threshold*next best
```

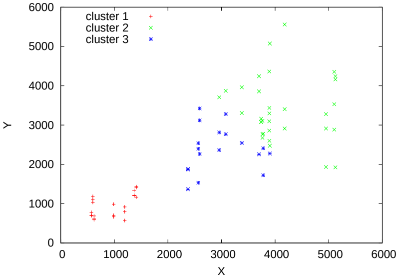
today's exercise: k-means clustering

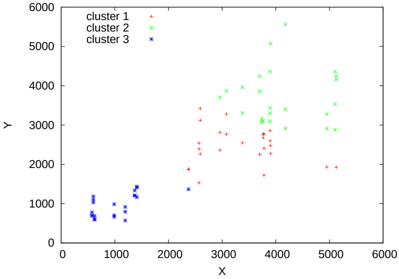
```
% ruby k-means.rb km-data.txt > km-results.txt
```



k-means clustering results

- different results by different initial values





k-means code (1/2)

```
k = 3 # k clusters
re = /^(\d+)\s+(\d+)/
INFINITY = 0x7fffffff

# read data
nodes = Array.new # array of array for data points: [x, y, cluster_index]
centroids = Array.new # array of array for centroids: [x, y]
ARGF.each_line do |line|
  if re.match(line)
    c = rand(k) # randomly assign initial cluster
    nodes.push [$1.to_i, $2.to_i, c]
  end
end

round = 0
begin
  updated = false

  # assignment step: assign each node to the closest centroid
  if round != 0  # skip assignment for the 1st round
    nodes.each do |node|
      dist2 = INFINITY # square of dsistance to the closest centroid
      cluster = 0 # closest cluster index
      for i in (0 .. k - 1)
        d2 = (node[0] - centroids[i][0])**2 + (node[1] - centroids[i][1])**2
        if d2 < dist2
          dist2 = d2
          cluster = i
        end
      end
      node[2] = cluster
    end
  end
```

k-means code (2/2)

```
  # update step: compute new centroids
  sums = Array.new(k)
  clsize = Array.new(k)
  for i in (0 .. k - 1)
    sums[i] = [0, 0]
    clsize[i] = 0
  end
  nodes.each do |node|
    i = node[2]
    sums[i][0] += node[0]
    sums[i][1] += node[1]
    clsize[i] += 1
  end

  for i in (0 .. k - 1)
    newcenter = [Float(sums[i][0]) / clsize[i], Float(sums[i][1]) / clsize[i]]
    if round == 0 || newcenter[0] != centroids[i][0] || newcenter[1] != centroids[i][1]
      centroids[i] = newcenter
      updated = true
    end
  end

  round += 1

end while updated == true

# print the results
nodes.each do |node|
  puts "#{node[0]}\t#{node[1]}\t#{node[2]}"
end
```

gnuplot script

```
set key left
set xrange [0:6000]
set yrange [0:6000]
set xlabel "X"
set ylabel "Y"

plot "km-results.txt" using 1:($3==0?$2:1/0) title "cluster 1" with points, \
"km-results.txt" using 1:($3==1?$2:1/0) title "cluster 2" with points, \
"km-results.txt" using 1:($3==2?$2:1/0) title "cluster 3" with points
```

summary

Class 11 Data Mining

- Pattern extraction
- Classification
- Clustering
- exercise: clustering

next class

Class 12 Search and Ranking (12/19)

- Search systems
- PageRank
- exercise: PageRank algorithm
- **on final report**