Internet Measurement and Data Analysis (6)

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review of previous class

Class 5 Diversity and complexity (10/30)

- Long tail
- Web access and content distribution
- Power-law and complex systems
- exercise: power-law analysis

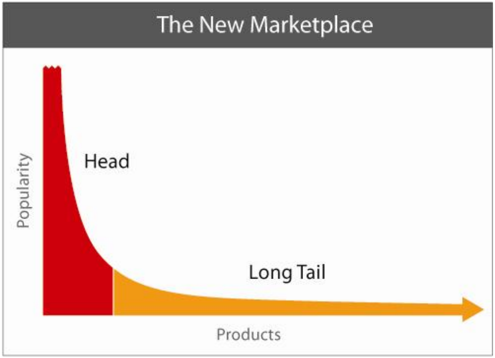
today's topics

Class 6 Correlation

- Online recommendation systems
- Distance
- Correlation coefficient
- exercise: correlation analysis

online recommender systems

- finding potential needs for long-tail users at EC sites
  - by recommending products which fit each user's taste
- widely used as the cost goes down by recomender package software



source: `http://longtail.com/`

recommender systems

- from user online behavior, infer useful information for users automatically
- EC sites: recommend products automatically from purchase or view records
- other applications: music, movies, search engine, etc

different approaches for database structure

- item based: compile data for each item
- user based: compile data for each user
- most systems combine both

prediction methods of recommender systems

- content based:
  - recommend items similar to the items the user used in the past
    - (manual) classifications of items
    - clustering items by machine learning methods
    - building rules from know-how
  - tend to recommend items in the same group, less surprising
- collaborative filtering: employed by amazon and others
  - e.g., "users who bought X also bought Y"
  - compute similarities among users from their online activities
  - recommend items bought by similar users
  - main feature: it does not use the information about items
  - could lead to surprising findings for user (serendipity)
- naive bayesian filter: often used for spam filtering
  - machine-learning technique to compute probabilities from a large number of item and user attributes

recent advances in targeted advertising

- targeted advertising
  - advertisements intended to reach specific consumer groups
  - so as to improve the effectiveness and cost-benefit
- online advertising networks
  - web services that connect advertisers to web publishers
  - e.g., a banner advertisement at a personal web site
- Real Time Bidding
  - platform for real-time auction of online advertisements
  - web publishers offer display space on user's visit
    - with user's attributes and activity history (tracked by cookies)
  - bid managers provides a platform for auction
  - advertisers place a bid for advertisement
    - decide the price based on the provided information
    - retargetting: for users who visited the advertising company in the past
  - RTB auction process completes in less than 100ms

collaborative filtering

- several well-known algorithms
- example: simple correlation analysis between users
  - compute correlation between users to find similar users
  - rate item as a sum of others' scores weighted by the similarity

example: purchase history

	item						
user	a	b	c	d	e	f	...
A	1		1		1		...
B			1	1			...
C	1	1					...
D	1		1		1		...
...							...

compute the scores of items that A does not have but A's similar users have

	similarity	item						
user	σ	a	b	c	d	e	f	...
A	1	1		1		1		...
S	0.88		0.88		-		0.88	...
C	0.81		0.81		-		-	...
K	0.75		-		-		-	...
F	0.73		0.73		0.73		0.73	...
score			2.50		0.73		1.61	...

Example: Netflix Prize

- an open annual competition for collaborative filtering algorithms to predict user ratings for movies
- sponsored by Netflix, an online DVD-rental/download service company
- competition: data set $< user\_id, movie\_id, date\_of\_grade, grade >$
  - training data set (100 million ratings)
  - qualifying data set (2.8 million ratings)
    - quiz data set (1.4 million)
    - test data set (1.4 million)
  - results are scored by root mean squared error
- competition started in 2006 and ended in 2009
  - criticized by privacy advocates

distances

various distances

- Euclidean distance
- standardized Euclidean distance
- Minkowski distance
- Mahalanobis distance

similarities

- binary vector similarities
- n-dimensional vector similarities

properties of distance

a metric of distance $d(x, y)$ between 2 points (x, y) in space
positivity

$$d(x, y) \geq 0$$

$$d(x, y) = 0 \Leftrightarrow x = y$$

symmetry

$$d(x, y) = d(y, x)$$

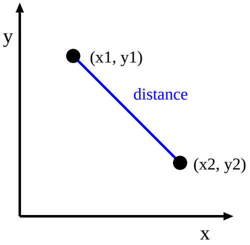
triangle inequality

$$d(x, z) \leq d(x, y) + d(y, z)$$

Euclidean distance

word “distance” usually means “Euclidean distance”
a distance of 2 points (x, y) in a n-dimensional space

$$d(x, y) = \sqrt{\sum_{k=1}^{n} (x_k - y_k)^2}$$

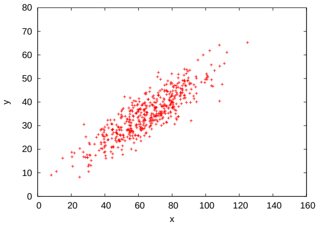


euclidean distance in 2-dimensional space

standardized Euclidean distance

- when variances are different among variables, distances are affected.
- standard Euclidean distance: normalized by dividing the Euclidean distance by the variance of each variable

$$d(x,y) = \sqrt{\sum_{k=1}^{n} (\frac{x_k}{s_k} - \frac{y_k}{s_k})^2} = \sqrt{\sum_{k=1}^{n} \frac{(x_k - y_k)^2}{s_k^2}}$$

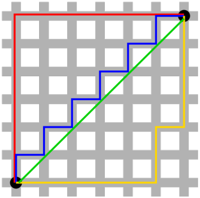


Minkowski distance

generalization of Euclidean distance: as parameter r grows, a short cut crossing different axes is preferred more

$$d(x, y) = (\sum_{k=1}^{n} |x_k - y_k|^r)^{\frac{1}{r}}$$

- $r = 1$: Manhattan distance
  - Hamming distance: for 2 strings of equal length, the number of positions at which the corresponding symbols are different.
  - example: the hamming distance of 111111 and 101010 is 3
- $r = 2$: Euclidean distance



Manhattan distance vs. Euclidean distance

vector norm (1/2)

vector norm: the length of a vector

$$\|x\| \text{ where } x \text{ is a vector}$$

the l_n-norm of x is defined by Minkowski distance as

$$\|x\|_n = \sqrt[n]{\sum_i |x_i|^n}$$

l_0-norm: the total number of non-zero elements in a vector

$$\|x\|_0 = \#(i|x_i \neq 0)$$

l_1-norm: sum of absolute difference

$$\|x\|_1 = \sum_i |x_i|$$

l_2-norm: Euclidean distance

$$\|x\|_2 = \sqrt{\sum_i |x_i|^2}$$

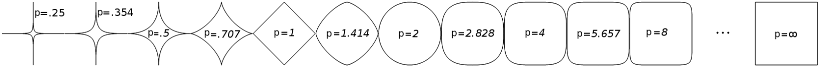
l_∞-norm: the maximum entry's magnitude of a vector

$$\|x\|_\infty = max(|x_i|)$$

vector norm (2/2)

For the example vector $x = (1, 2, 3)$

$$
\begin{array}{rc}
\|x\|_0 & 3 = 3.000 \\
\|x\|_1 & 6 = 6.000 \\
\|x\|_2 & \sqrt{14} = 3.742 \\
\|x\|_3 & 6^{2/3} = 3.302 \\
\|x\|_4 & 2^{1/4}\sqrt{7} = 3.146 \\
\|x\|_\infty & 3 = 3.000
\end{array}
$$



unit circles of l_p-norm with various values of p

Mahalanobis distance

a distance that takes correlations into account, when correlation exists between variables

$$mahalanobis(x, y) = (x - y)\Sigma^{-1}(x - y)^T$$

here, Σ^{-1} is the inverse matrix of its covariance matrix

similarities

similarity

- numerical measure of how alike 2 data objects are

properties of similarity

positivity

$$0 \leq s(x, y) \leq 1$$

$$s(x, y) = 1 \Leftrightarrow x = y$$

symmetry

$$s(x, y) = s(y, x)$$

in general, triangle inequality does not apply to similarities

similarity between binary vectors

Jaccard coefficient

- used for similarity between binary vectors in which the occurrences of 1 is much smaller than the occurrences of 0
- example: as a metric of similarity by occurrences of words in documents
- many words do not appear in both documents ⇒ not considered
- the following table shows the relationship of each item

		vector y	
		1	0
vector x	1	n_{11}	n_{10}
	0	n_{01}	n_{00}

Jaccard coefficient:

$$J = \frac{n_{11}}{n_{11} + n_{10} + n_{01}}$$

similarity between vectors

similarity between (non-binary) vectors

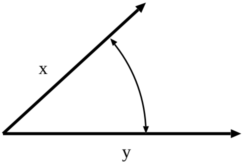
- example: similarity of documents where frequencies of words are also taken into consideration

cosine similarity

- take the angle (cosine) of (x, y) of vectors
- normalized by the length of the vector $\Rightarrow$ length is not considered

$$cos(x, y) = \frac{x \cdot y}{\|x\| \|y\|}$$

$x \cdot y = \sum_{k=1}^{n} x_k y_k$: product of vectors
$\|x\| = \sqrt{\sum_{k=1}^{n} x_k^2} = \sqrt{x \cdot x}$: length of the vector



example: cosine similarity

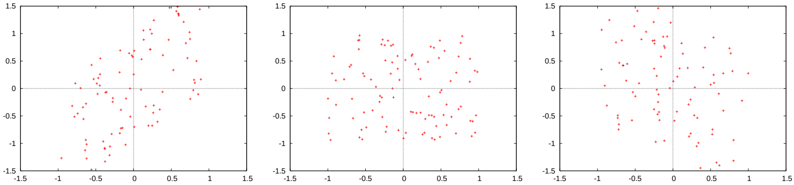
$$x = 3 \quad 2 \quad 0 \quad 5 \quad 0 \quad 0 \quad 0 \quad 2 \quad 0 \quad 0$$
$$y = 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 2$$

$$x \cdot y = 3 * 1 + 2 * 1 = 5$$
$$\|x\| = \sqrt{3 * 3 + 2 * 2 + 5 * 5 + 2 * 2} = \sqrt{42} = 6.481$$
$$\|y\| = \sqrt{1 * 1 + 1 * 1 + 2 * 2} = \sqrt{6} = 2.449$$

$$cos(x, y) = \frac{5}{6.481 * 2.449} = 0.315$$

scatter plots and correlation

- explores relationships between 2 variables
  - X-axis: variable X
  - Y-axis: corresponding value of variable Y
- you can identify
  - whether variables X and Y related
    - no relation, positive correlation, negative correlation
- correlation coefficient: a measure of the strength and direction of correlation



examples: positive correlation 0.7 (left), no correlation 0.0 (middle), negative correlation -0.5 (right)

correlation

- covariance:

$$\sigma_{xy}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

- correlation coefficient:

$$\rho_{xy} = \frac{\sigma_{xy}^2}{\sigma_x \sigma_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

- correlation coefficient: the covariance of 2 variables normalized by their product of their standard deviations, a value between -1 and $+1$ inclusive.
- sensitive to outliers. so, you should use a scatter plot to observe outliers.
- correlation and causality
  - correlation does not imply causal relationship
    - third factor C causes both A and B (e.g., break and test score)
    - coincidence

computing correlation coefficient (1)

sum of squares

$$\begin{aligned}\sum_{i=1}^{n}(x_i - \bar{x})^2 &= \sum_{i=1}^{n}(x_i^2 - 2x_i\bar{x} + \bar{x}^2) \\ &= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + n\,\bar{x}^2 \\ &= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\cdot n\,\bar{x} + n\,\bar{x}^2 \\ &= \sum_{i=1}^{n} x_i^2 - n\,\bar{x}^2 = \sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}\end{aligned}$$

sum of products

$$\begin{aligned}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) &= \sum_{i=1}^{n}(x_i y_i - x_i\bar{y} - \bar{x}y_i + \bar{x}\bar{y}) \\ &= \sum_{i=1}^{n} x_i y_i - \bar{x}\sum_{i=1}^{n} y_i - \bar{y}\sum_{i=1}^{n} x_i + n\,\bar{x}\bar{y} \\ &= \sum_{i=1}^{n} x_i y_i - \bar{x}\cdot n\,\bar{y} - \bar{y}\cdot n\,\bar{x} + n\,\bar{x}\bar{y} \\ &= \sum_{i=1}^{n} x_i y_i - n\,\bar{x}\bar{y} = \sum_{i=1}^{n} x_i y_i - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}\end{aligned}$$

computing correlation coefficient (2)

correlation coefficient

$$\begin{aligned}
\rho_{xy} &= \frac{\sigma_{xy}^2}{\sigma_x \sigma_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} \\
&= \frac{\sum_{i=1}^{n} x_i y_i - n\,\bar{x}\bar{y}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - n\,\bar{x}^2)(\sum_{i=1}^{n} y_i^2 - n\,\bar{y}^2)}} \\
&= \frac{\sum_{i=1}^{n} x_i y_i - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n})(\sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n})}}
\end{aligned}$$

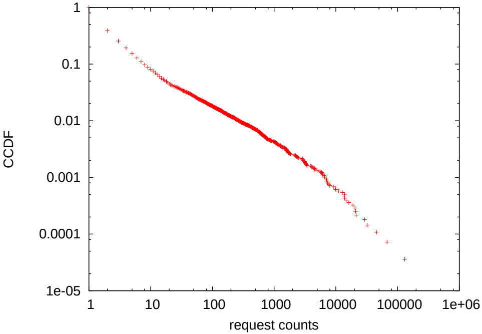
other correlation coefficients

- Pearson's product-moment correlation coefficient
  - or simply "correlation coefficient" (what we have learned)
- rank correlation coefficient: relationships between different rankings on the same set of items
  - Spearman's rank correlation coefficient
  - Kendall's rank correlation coefficient
- others

previous exercise: CCDF plots

extract the access count of each unique content from the JAIST server access log, plot the access count distribution in CCDF

```
% ./count_contents.rb sample_access_log > contents.txt
% ./make_ccdf.rb contents.txt > ccdf.txt
```



extracting the access count of each unique content

```
# output: URL req_count byte_count
# regular expression for apache combined log format
#   host ident user time request status bytes referer agent
re = /^(\S+) (\S+) (\S+) \[(.*?)\] "(.*?)" (\d+) (\d+|-) "(.*?)" "(.*?)"/
# regular expression for request: method url proto
req_re = /(\w+) (\S+) (\S+)/
contents = Hash.new([0, 0])
count = parsed = 0
ARGF.each_line do |line|
  count += 1
  if re.match(line)
    host, ident, user, time, request, status, bytes, referer, agent = $~.captures
    # ignore if the status is not success (2xx)
    next unless /2\d{2}/.match(status)
    if req_re.match(request)
      method, url, proto = $~.captures
      # ignore if the method is not GET
      next unless /GET/.match(method)
      parsed += 1
      # count contents by request and bytes
      contents[url] = [contents[url][0] + 1, contents[url][1] + bytes.to_i]
    else
      # match failed.  print a warning msg
      $stderr.puts("request match failed at line #{count}: #{line.dump}")
    end
  else
    $stderr.puts("match failed at line #{count}: #{line.dump}") # match failed.
  end
end
contents.sort_by{|key, value| -value[0]}.each do |key, value|
  puts "#{key} #{value[0]} #{value[1]}"
end
$stderr.puts "# #{contents.size} unique contents in #{parsed} successful GET requests"
$stderr.puts "# parsed:#{parsed} ignored:#{count - parsed}"
```

access count of each unique content

```
% cat contents.txt
/project/linuxonandroid/Ubuntu/12.04/full/ubuntu1204-v4-full.zip 25535 17829045
/project/morefont/xiongmaozhongwen.apk 10949 13535294486
/project/morefont/zhongguoxin.apk 9047 9549531354
/project/honi/some_software/Windows/Office_Plus_2010_SP1_W32_xp911.com.rar 5616
/project/morefont/fangzhengyouyijian.apk 5609 2879391721
/pub/Linux/CentOS/5.9/extras/i386/repodata/repomd.xml 5121 12213484
/pub/Linux/CentOS/5.9/updates/i386/repodata/repomd.xml 5006 10969621
/pub/Linux/CentOS/5.9/os/i386/repodata/repomd.xml 4953 6832653
/project/nppluginmgr/xml/plugins.md5.txt 4881 1369547
/project/winpenpack/X-LenMus/releases/X-LenMus_5.3.1_rev5.zip 4689 990250462

...

/pub/Linux/openSUSE/distribution/12.3/repo/oss/suse/x86_64/gedit-3.6.2-2.1.2.x8
/pub/sourceforge/n/nz/nzbcatcher/source/?C=D;O=A 1 1075
/ubuntu/pool/universe/m/mmass/mmass_5.4.1.orig.tar.gz 1 3754849
```

script to convert the access count to CCDF

```
#!/usr/bin/env ruby

re = /^\S+\s+(\d+)\s+\d+/

n = 0
counts = Hash.new(0)
ARGF.each_line do |line|
  if re.match(line)
    counts[$1.to_i] += 1
    n += 1
  end
end

cum = 0
counts.sort.each do |key, value|
  comp = 1.0 - Float(cum) / n
  puts "#{key} #{value} #{comp}"
  cum += value
end
```

cumulative access counts

```
% cat ccdf.txt
1 84414 1.0
2 9813 0.2315731022366253
3 5199 0.14224463601358184
4 3034 0.0949177537254331
5 1636 0.06729902688137779
6 1083 0.05240639764048316
7 663 0.04254776838138241
8 495 0.03651243024769468
9 367 0.03200640856417214
10 274 0.028665580366489807

...

5616 1 3.6412296432475344e-05
9047 1 2.730922232441202e-05
10949 1 1.8206148216237672e-05
25535 1 9.103074108174347e-06
```

gnuplot script for plotting the content access count in CCDF

```
set logscale
set xlabel "request counts"
set ylabel "CCDF"

plot "ccdf.txt" using 1:3 notitle with points
```

assignment 1: the finish time distribution of a marathon

- purpose: investigate the distribution of a real-world data set
- data: the finish time records from honolulu marathon 2012
  - http://results.sportstats.ca/res2012/honolulumarathon_m.htm
  - the number of finishers: 24,070
- items to submit
  1. mean, standard deviation and median of the total finishers, male finishers, and female finishers
  2. the distributions of finish time for each group (total, men, and women)
     - plot 3 histograms for 3 groups
     - use 10 minutes for the bin size
     - use the same scale for the axes to compare the 3 plots
  3. CDF plot of the finish time distributions of the 3 group
     - plot 3 groups in a single graph
  4. discuss differences in finish time between male and female. what can you observe from the data?
  5. optional
     - other analysis of your choice (e.g., discussion on differences among age groups)
- submission format: a single PDF file including item 1-5
- submission method: upload the PDF file through SFC-SFS
- submission due: 2013-11-07

honolulu marathon data set

data format

```
      Chip     Pace                                                      Gender    Category        @10km   @21.1   @3
Place Time     /mi      #    Name                City         ST CNT Plce/Tot Plc/Tot Category Split1 Split2
---- -------- ---- ----- ------------------- ------------ -- --- -------- ------- ------ ----- ------- ---
   1 02:12:31 5:04     6 Kipsang, Wilson     Iten             KEN  1/12690  1/16   MElite 31:40 1:07:07 1:3
   2 02:13:08 5:05     7 Geneti, Markos      Addis Ababa      ETH  2/12690  2/16   MElite 31:39 1:07:02 1:3
   3 02:14:15 5:08    11 Kimutai, Kiplimo    Eldoret          KEN  3/12690  3/16   MElite 31:40 1:07:02 1:3
   4 02:14:55 5:09     2 Ivuti, Patrick      Kangundo         KEN  4/12690  4/16   MElite 31:40 1:07:02 1:3
   5 02:15:17 5:10    12 Arile, Julius       Kepenguria       KEN  5/12690  5/16   MElite 31:39 1:07:02 1:3
   6 02:15:53 5:11     9 Bouramdane, Abderr  Champs De Cou    MAR  6/12690  6/16   MElite 31:40 1:07:01 1:3
   7 02:18:27 5:17     8 Manza, Nicholas     Ngong Hills      KEN  7/12690  7/16   MElite 31:39 1:07:01 1:3
   8 02:19:46 5:20     1 Chelimo, Nicholas   Ngong Hills      KEN  8/12690  8/16   MElite 31:40 1:07:02 1:3
   9 02:25:23 5:33 20850 Harada, Taku        Nagoya-Shi    AI JPN  9/12690  1/1238 M25-29 31:54 1:09:52 1:4
  10 02:27:12 5:37 25474 Hagawa, Eiichi      Matsumoto     NA JPN 10/12690  1/1501 M30-34 32:46 1:12:21 1:4
...
```

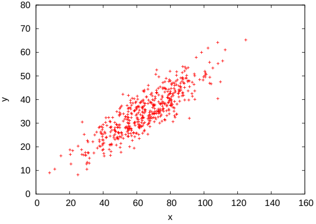
- Chip Time: finish time
- Category: MElite, WElite, M15-19, M20-24, ..., W15-29, W20-24, ...
  - note some runners have "No Age" for Category
- Country: 3-letter country code: e.g., JPN, USA
  - note some runners have "UK" for country-code
- check the number of the total finishers when you extract the finishers

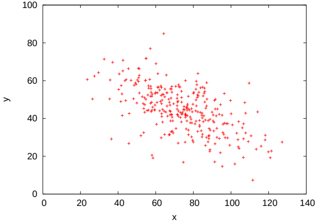
today's exercise: computing correlation coefficient

- compute correlation coefficient using the sample data sets
  - correlation-data-1.txt, correlation-data-2.txt

correlation coefficient

$$\rho_{xy} = \frac{\sigma_{xy}^2}{\sigma_x \sigma_y} = \frac{\sum_{i=1}^{n} x_i y_i - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n})(\sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n})}}$$





data-1:r=0.87 (left), data-2:r=-0.60 (right)

script to compute correlation coefficient

```
#!/usr/bin/env ruby

# regular expression for matching 2 floating numbers
re = /([-+]?\d+(?:\.\d+)?)\s+([-+]?\d+(?:\.\d+)?)/

sum_x = 0.0 # sum of x
sum_y = 0.0 # sum of y
sum_xx = 0.0 # sum of x^2
sum_yy = 0.0 # sum of y^2
sum_xy = 0.0 # sum of xy
n = 0 # the number of data

ARGF.each_line do |line|
    if re.match(line)
      x = $1.to_f
      y = $2.to_f
      sum_x += x
      sum_y += y
      sum_xx += x**2
      sum_yy += y**2
      sum_xy += x * y
      n += 1
    end
end

r = (sum_xy - sum_x * sum_y / n) /
  Math.sqrt((sum_xx - sum_x**2 / n) * (sum_yy - sum_y**2 / n))

printf "n:%d r:%.3f\n", n, r
```

summary

Class 6 Correlation

- Online recommendation systems
- Distance
- Correlation coefficient
- exercise: correlation analysis

next class

Class 7 Multivariate analysis (11/13)

- Data sensing and GeoLocation
- Linear regression
- Principal Component Analysis
- exercise: linear regression