Internet Measurement and Data Analysis (4)

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review of previous class

Class 3 Data recording and log analysis (10/20)

- Data format
- Log analysis methods
- exercise: log data and regular expression

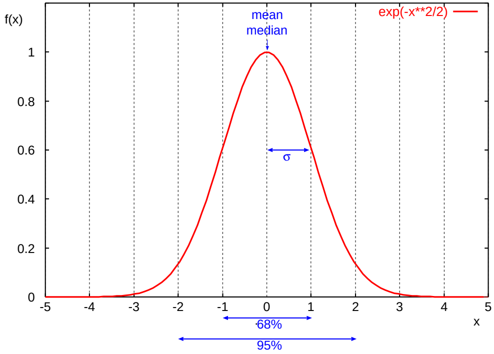
today's topics

Class 4 Distribution and confidence intervals

- Normal distribution
- Confidence intervals and statistical tests
- Distribution generation
- exercise: confidence intervals
- **assignment 1**

normal distribution (1/2)

- also known as gaussian distribution
- $N(\mu, \sigma)$: defined by 2 parameters: μ:mean, σ:standard deviation
- sum of random variables follows normal distribution
- standard normal distribution: $\mu = 0, \sigma = 1$
- in normal distribution
  - 68% within $(mean - stddev, mean + stddev)$
  - 95% within $(mean - 2 * stddev, mean + 2 * stddev)$



normal distribution (2/2)

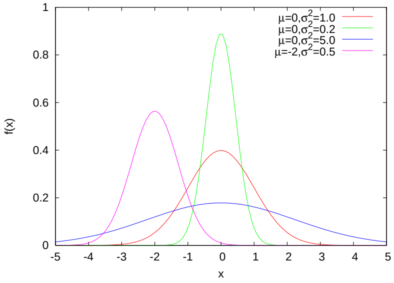
probability density function (PDF)

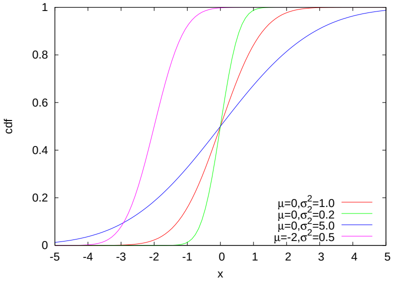
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

cumulative distribution function (CDF)

$$F(x) = \frac{1}{2}(1 + erf\frac{x-\mu}{\sigma\sqrt{2}})$$

$$\mu : mean, \sigma : standarddeviation$$





confidence interval

- confidence interval
  - provides probabilistic bounds
  - tells how much uncertainty in the estimate
- confidence level, significance level

$$Prob\{c_1 \leq \mu \leq c_2\} = 1 - \alpha$$

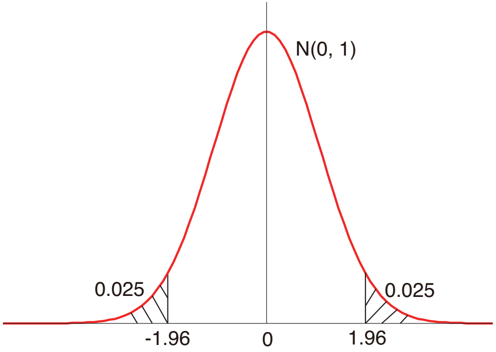
$$\begin{array}{ll} (c1, c2): & confidence\ interval \\ 100(1-\alpha): & confidence\ level \\ \alpha: & significance\ level \end{array}$$

- example: with 95% confidence, the population mean is between $c1$ and $c2$
- traditionally, 95% and 99% are often used for confidence level

95% confidence interval

sample mean from normal distribution $N(\mu, \sigma)$ follows normal distribution $N(\mu, \sigma/\sqrt(n))$
95% confidence interval corresponds to the following area in the standard normal distribution

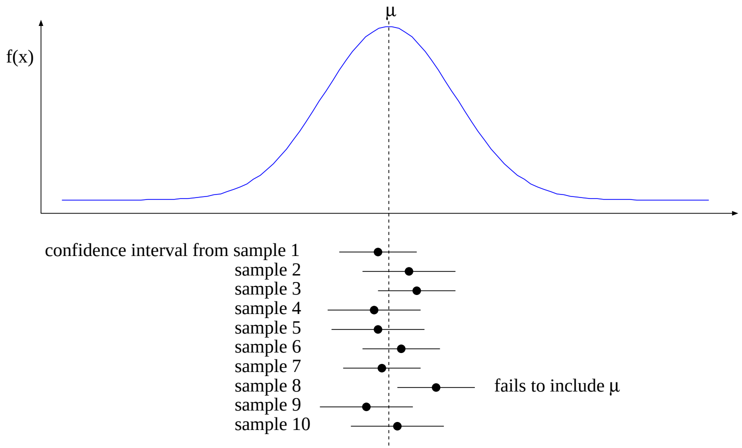
$$-1.96 \le \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \le 1.96$$



standard normal distribution N(0, 1)

illustration of confidence interval

- confidence level 90% means 90% samples will contain population mean in their confidence intervals



confidence interval for mean

when sample size is large, confidence interval for population mean is

$$\bar{x} \mp z_{1-\alpha/2}\, s/\sqrt{n}$$

here, $\bar{x}$:sample mean, s:sample standard deviation, n:sample size,α:significance level

$z_{1-\alpha/2}$:$(1-\alpha/2)$-quantile of unit normal variate

- for 95% confidence level: $z_{1-0.05/2} = 1.960$
- for 90% confidence level: $z_{1-0.10/2} = 1.645$
- example: 5 measurements of TCP throughput
  - 3.2, 3.4, 3.6, 3.6, 4.0Mbps
  - sample mean $\bar{x} = 3.56$Mbps, sample standard deviation $s = 0.30$Mbps
  - 95% confidence interval:

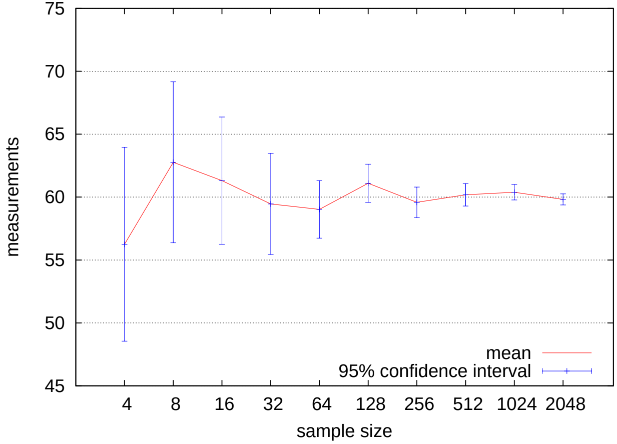
$$\bar{x} \mp 1.96(s/\sqrt{n}) = 3.56 \mp 1.960 \times 0.30/\sqrt{5} = 3.56 \mp 0.26$$

  - 90% confidence interval:

$$\bar{x} \mp 1.645(s/\sqrt{n}) = 3.56 \mp 1.645 \times 0.30/\sqrt{5} = 3.56 \mp 0.22$$

confidence interval for mean and sample size

confidence interval becomes smaller as sample size increases



confidence interval with varying sample size

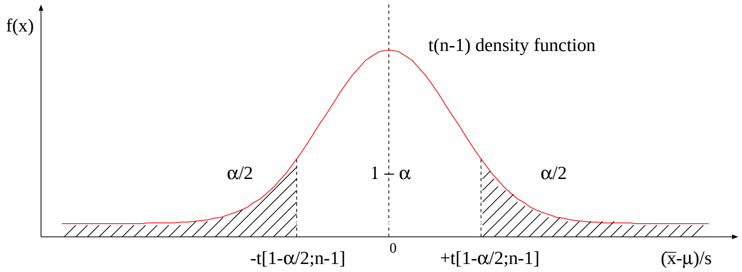
confidence interval for mean when sample size is small

when sample size is small (< 30), confidence interval can be constructed only if population has normal distribution

- $(\bar{x} - \mu)/(s/\sqrt{n})$ for samples from normal population follows $t(n-1)$ distribution

$$\bar{x} \mp t_{[1-\alpha/2;n-1]}\, s/\sqrt{n}$$

here, $t_{[1-\alpha/2;n-1]}$:$(1-\alpha/2)$-quantile of a t-variate with $(n-1)$ degree of freedom



example: confidence interval for mean when sample size is small

- example: in the previous TCP throughput measurement, confidence interval should be calculated using $t(n-1)$ distribution
  - 95% confidence interval, $n = 5$: $t_{[1-0.05/2,4]} = 2.776$

    $$\bar{x} \mp 2.776(s/\sqrt{n}) = 3.56 \mp 2.776 \times 0.30/\sqrt{5} = 3.56 \mp 0.37$$

  - 90% confidence interval, $n = 5$: $t_{[1-0.10/2,4]} = 2.132$

    $$\bar{x} \mp 2.132(s/\sqrt{n}) = 3.56 \mp 2.132 \times 0.30/\sqrt{5} = 3.56 \mp 0.29$$

other confidence intervals

- for population variance
  - chi-square distribution with degree of freedom $(n-1)$
- for ratio of sample variances
  - F distribution with degree of freedom (n_1-1, n_2-1)

how to use confidence interval

applications

- provide confidence interval to show possible range of mean
- from sample mean and stddev, compute how many trials are needed to satisfy a given confidence interval
- repeat measurement until a given confidence interval is reached

sample size for determining mean

- how many observations n is required to estimate population mean with accuracy ±r% and confidence level $100(1-\alpha)$%?
- perform preliminary test to obtain sample mean $\bar{x}$ and standard deviation s
- for sample size n, confidence interval is $\bar{x} \mp z\frac{s}{\sqrt{n}}$
- desired accuracy of r%

$$\bar{x} \mp z\frac{s}{\sqrt{n}} = \bar{x}(1 \mp \frac{r}{100})$$

$$n = (\frac{100zs}{r\bar{x}})^2$$

- example: by preliminary test for TCP throughput, the sample mean is 3.56Mbps, sample standard deviation is 0.30Mbps. how many observations will be required to obtain accuracy (< 0.1Mbps) with 95% confidence?

$$n = (\frac{100zs}{r\bar{x}})^2 = (\frac{100 \times 1.960 \times 0.30}{0.1/3.56 \times 100 \times 3.56})^2 = 34.6$$

inference and hypothesis testing

the purpose of hypothesis testing

- a method to statistically test a hypothesis on population using samples

inference and hypothesis testing: both sides of the coin

- inference: predict a value to be within a range
- hypothesis testing: whether a hypothesis is accepted or rejected
  - make a hypothesis about population, compute if the hypothesis falls within the 95% confidence interval
  - accept the hypothesis if it is within the range
  - reject the hypothesis if it is outside of the range

example: hypothesis testing

when flipping N coins, we have 10 heads. In this case, can we accept a hypothesis of $N = 36$? (here, assume the distribution follows normal distribution with $\mu = N/2, \sigma = \sqrt{n}/2$)

- hypothesis: 10 heads for $N = 36$
- hypothesis testing for 95% confidence level

$$-1.96 \leq (\bar{x} - 18)/3 \leq 1.96 \qquad 12.12 \leq \bar{x} \leq 23.88$$

10 is outside of the 95% confidence interval so that the hypothesis of $N = 36$ is rejected

discarding outliers

outliers should not be discarded blindly. investigation needed, which sometimes leads to new findings

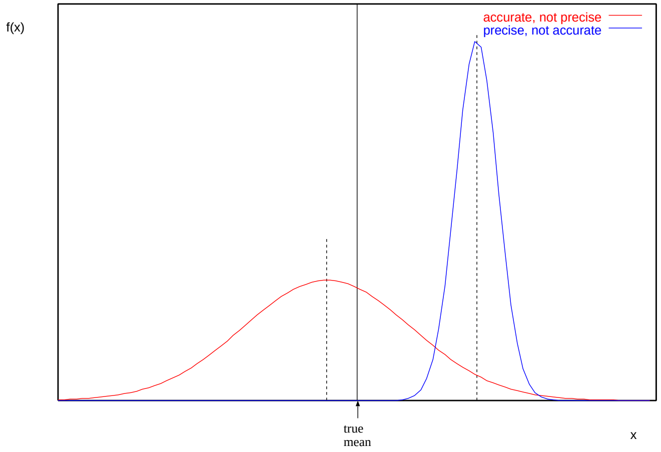
- Chauvenet's criterion: heuristic method to reject outliers
  - calculate sample mean and standard deviation from sample size n
  - assuming normal distribution, determine the probability p of suspected data point
  - if $n \times p < 0.5$, the suspicious data point may be discarded
  - note: when $n < 50$, s is not reliable. the method should not apply repeatedly
- example: 10 delay measurements: 4.6, 4.8, 4.4, 3.8, 4.5, 4.7, 5.8, 4.4, 4.5, 4.3 (sec). is it ok to discard 5.8sec?
  - $\bar{x} = 4.58, s = 0.51$
  - $t_{sus} = \frac{x_{sus} - \bar{x}}{s} = \frac{5.8-4.58}{0.51} = 2.4$, 2.4 times larger than s
  - $P(|x - \bar{x}| > 2.4s) = 1 - P(|x - \bar{x}| < 2.4s) = 1 - 0.984 = 0.016$
  - $n \times p = 10 \times 0.016 = 0.16$
  - $0.16 < 0.5$: we may discard 5.8sec

accuracy, precision and errors

accuracy: how close to true value

precision: uncertainty in data

error: difference from true value, range of uncertainty



various errors

measurement errors

- systematic errors (if conditions are identified, errors could be corrected)
  - instrument error, procedural error, personal bias
- random errors (noise: accuracy can be improved by repeating measurement)

calculation errors

- round-off errors
- truncation errors
- loss of trailing digits
- cancellation of significant digits
- propagation of error

sampling errors

- when sampling is used, true value is usually unknown
- sampling errors: errors in estimating population characteristics

significant digits

significant digits of "1.23" is 3 ($1.225 \leq 1.23 < 1.235$)

expressions

expressions	significant digits	
12.3	3	
12.300	5	
0.0034	2	
1200	4	(vague, 1.200×10^3)
2.34×10^4	3	

arithmetic

- use all the available digits during calculation
  - for manual calculation, use one more digit
- apply the significant digits to the final value

basic rules

- addition/subtraction: use the smallest number of decimal places
  - $1.23 + 5.724 = 6.954 \Rightarrow 6.95$
- multiplication/division: use the smallest number of significant digits
  - $4.23 \times 0.38 = 1.6074 \Rightarrow 1.6$

computational precision of computers

- integer (32/64bits)
  - 32bit signed integer (up to 2G)
- 32bit floating point (IEEE 754 single precision): significant digits:7
  - sign:1bit, exponent:8bits, mantissa:23bits
  - $16,000,000 + 1 = 16,000,000!!$
- 64bit floating point (IEEE 754 double precision): significant digits:15
  - sign:1bit, exponent:11bits, mantissa:52bits

previous exercise: web access log sample data

- apache log (combined log format)
- from a JAIST server, access log for 24 hours
- about 20MB (zip compressed), about 162MB after unzip
- 1/10 sampling
- client IP addresses are anonymized for privacy
    - using “ipv6loganon –anonymize-careful”

```
access log for 24 hours:
 http://www.iijlab.net/~kjc/classes/sfc2014f-measurement/sample_access_log.zip
```

sample data

```
117.136.16.0 - - [01/Oct/2013:23:59:58 +0900] "GET /project/morefont/liangqiushengshufaziti.apk \
  HTTP/1.1" 200 524600 "-" "-" jaist.dl.sourceforge.net
218.234.160.0 - - [01/Oct/2013:23:59:59 +0900] "GET /pub/Linux/linuxmint/packages/dists/olivia/\
  upstream/i18n/Translation-ko.xz HTTP/1.1" 404 564 "-" "Debian APT-HTTP/1.3 (0.9.7.7ubuntu4)" \
  ftp.jaist.ac.jp
119.80.32.0 - - [01/Oct/2013:23:59:59 +0900] "GET /project/morefont/xiongtuti.apk HTTP/1.1" 304 \
  132 "-" "Mozilla/4.0 (compatible; MSIE 6.0; Windows NT 5.1; Foxy/1; InfoPath.1)" \
  jaist.dl.sourceforge.net
218.234.160.0 - - [02/Oct/2013:00:00:00 +0900] "GET /pub/Linux/linuxmint/packages/dists/olivia/\
  import/i18n/Translation-en.gz HTTP/1.1" 404 562 "-" "Debian APT-HTTP/1.3 (0.9.7.7ubuntu4)" \
  ftp.jaist.ac.jp
117.136.0.0 - - [02/Oct/2013:00:00:00 +0900] "GET /project/morefont/xiaoqingwaziti.apk HTTP/1.1"\
  200 590136 "-" "-" jaist.dl.sourceforge.net
123.224.224.0 - - [02/Oct/2013:00:00:00 +0900] "GET /pub/Linux/ubuntu/dists/raring/main/i18n/\
  Translation-en.bz2 HTTP/1.1" 304 187 "-" "Debian APT-HTTP/1.3 (0.9.7.7ubuntu4)" ftp.jaist.ac.jp
123.224.224.0 - - [02/Oct/2013:00:00:00 +0900] "GET /pub/Linux/ubuntu/dists/raring/multiverse/\
  i18n/Translation-en.bz2 HTTP/1.1" 304 186 "-" "Debian APT-HTTP/1.3 (0.9.7.7ubuntu4)" \
  ftp.jaist.ac.jp
124.41.64.0 - - [01/Oct/2013:23:59:58 +0900] "GET /ubuntu/pool/universe/s/shorewall6/\
  shorewall6_4.4.26.1-1_all.deb HTTP/1.1" 200 435975 "-" "Wget/1.14 (linux-gnu)" ftp.jaist.ac.jp
...
240b:10:c140:a909:a949:4291:c02d:5d13 - - [02/Oct/2013:00:00:01 +0900] "GET /ubuntu/pool/main/m/\
  manpages/manpages_3.52-1ubuntu1_all.deb HTTP/1.1" 200 626951 "-" \
  "Debian APT-HTTP/1.3 (0.9.7.7ubuntu4)" ftp.jaist.ac.jp
...
```

previous exercise: plotting request counts over time

- use the sample data
- extract request counts and transferred bytes with 5 minutes bins
- plot the results

```
% ruby parse_accesslog.rb sample_access_log > access-5min.txt
% more access-5min.txt
2013-10-01T20:00 1 1444348221
...
2013-10-01T23:55 215 1204698404
2013-10-02T00:00 2410 5607857319
2013-10-02T00:05 2344 3528532804
2013-10-02T00:10 2502 4354264670
2013-10-02T00:15 2555 5441105487
...
% gnuplot
gnuplot> load 'access.plt'
```

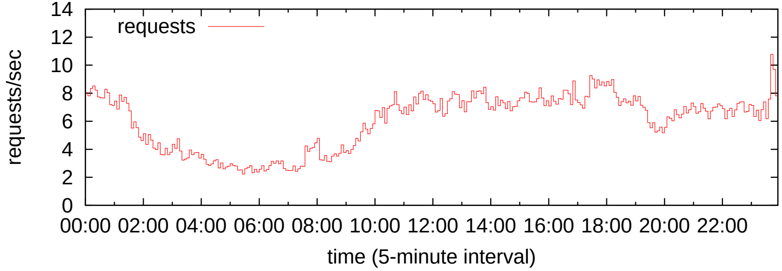
extract request counts and transferred bytes with 5 minutes bins

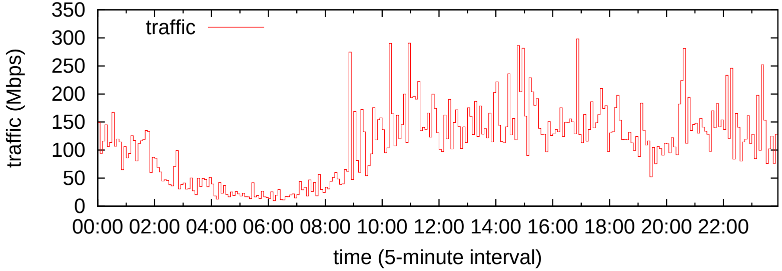
```
#!/usr/bin/env ruby
require 'date'

# regular expression for apache common log format
#   host ident user time request status bytes
re = /^(\S+) (\S+) (\S+) \[(.*?)\] "(.*?)" (\d+) (\d+|-)/
timebins = Hash.new([0, 0])
count = parsed = 0
ARGF.each_line do |line|
  count += 1
  if re.match(line)
    host, ident, user, time, request, status, bytes = $~.captures

    next unless request.match(/GET\s.*/) # ignore if the request is not "GET"
    next unless status.match(/2\d{2}/) # ignore if the status is not success (2xx)
    parsed += 1
    # parse timestamp
    ts = DateTime.strptime(time, '%d/%b/%Y:%H:%M:%S')
    # create the corresponding key for 5-minutes timebins
    rounded = sprintf("%02d", ts.min.to_i / 5 * 5)
    key = ts.strftime("%Y-%m-%dT%H:#{rounded}")
    # count by request and byte
    timebins[key] = [timebins[key][0] + 1, timebins[key][1] + bytes.to_i]
  else
    # match failed
    $stderr.puts("match failed at line #{count}: #{line.dump}")
  end
end
timebins.sort.each do |key, value|
  puts "#{key} #{value[0]} #{value[1]}"
end
$stderr.puts "parsed:#{parsed} ignored:#{count - parsed}"
```

plot graphs of request counts and transferred bytes





gnuplot script

- put 2 graphs together using multiplot

```
set xlabel "time (5-minute interval)"
set xdata time
set format x "%H:%M"
set timefmt "%Y-%m-%dT%H:%M"
set xrange ['2013-10-02T00:00':'2013-10-02T23:55']
set key left top

set multiplot layout 2,1

set yrange [0:14]
set ylabel "requests/sec"
plot "access-5min.txt" using 1:($2/300) title 'requests' with steps

set yrange [0:350]
set ylabel "traffic (Mbps)"
plot "access-5min.txt" using 1:($3*8/300/1000000) title 'traffic' with steps

unset multiplot
```

exercise: generating normally distributed random numbers

- generating pseudo random numbers that follow the normal distribution
  - write a program to generate normally distributed random numbers with mean u and standard deviation s, using a uniform random number generator function (e.g., rand in ruby)
- plotting a histogram
  - generate random numbers that follow the standard normal distribution, plot the histogram to confirm the standard normal distribution.
- computing confidence intervals
  - observe confidence interval changes according to sample size. use the normally distributed random number generator to produce 10 sets of normally distributed random numbers with mean 60 and standard deviation 10. sample size n = 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048
  - compute the confidence interval of the population mean from each sample set.
    use confidence level 95% and confidence interval "$\pm 1.960 \frac{s}{\sqrt{n}}$".
    plot the results of 10 sets in a single graph. plot sample size n on the X-axis in log-scale and mean and confidence interval on the Y-axis in linear scale

box-muller transform

basic form: creates 2 normally distributed random variables, z_0 and z_1, from 2 uniformly distributed random variables, u_0 and u_1, in $(0,1]$

$$z_0 = R\cos(\theta) = \sqrt{-2\ln u_0}\cos(2\pi u_1)$$
$$z_1 = R\sin(\theta) = \sqrt{-2\ln u_0}\sin(2\pi u_1)$$

polar form: approximation without trigonometric functions
u_0 and u_1: uniformly distributed random variables in $[-1,1]$,
$s = u_0^2 + u_1^2$ (if $s = 0$ or $s \geq 1$, re-select u_0, u_1)

$$z_0 = u_0\sqrt{\frac{-2\ln s}{s}}$$
$$z_1 = u_1\sqrt{\frac{-2\ln s}{s}}$$

random number generator code by box-muller transform

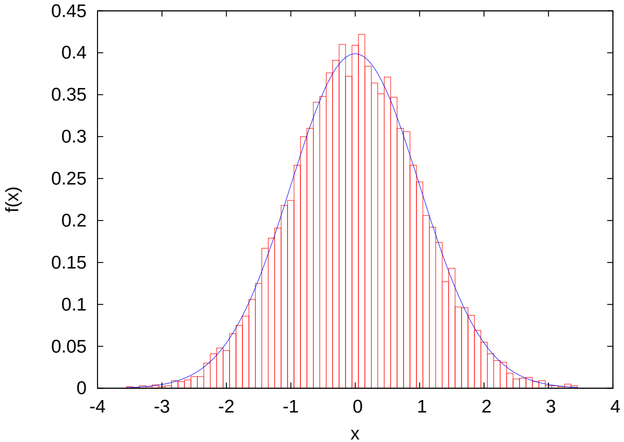
```
# usage: box-muller.rb [n [m [s]]]
n = 1 # number of samples to output
mean = 0.0
stddev = 1.0

n = ARGV[0].to_i if ARGV.length >= 1
mean = ARGV[1].to_i if ARGV.length >= 2
stddev = ARGV[2].to_i if ARGV.length >= 3

# function box_muller implements the polar form of the box muller method,
# and returns 2 pseudo random numbers from standard normal distribution
def box_muller
  begin
    u1 = 2.0 * rand - 1.0  # uniformly distributed random numbers
    u2 = 2.0 * rand - 1.0  # ditto
    s = u1*u1 + u2*u2    # variance
  end while s == 0.0 || s >= 1.0
  w = Math.sqrt(-2.0 * Math.log(s) / s) # weight
  g1 = u1 * w  # normally distributed random number
  g2 = u2 * w  # ditto
  return g1, g2
end
# box_muller returns 2 random numbers.  so, use them for odd/even rounds
x = x2 = nil
n.times do
  if x2 == nil
    x, x2 = box_muller
  else
    x = x2
    x2 = nil
  end
  x = mean + x * stddev  # scale with mean and stddev
  printf "%.6f\n", x
end
```

plot a histogram of normally distributed random numbers

- plot a histogram of random numbers following the standard normal distribution, and confirm that they are normally distributed
- generate 10,000 random numbers from the standard normal distribution, use bins with one decimal place for the histogram



plotting a histogram

- plot a histogram using bins with one decimal place

```
#
# create histogram: bins with 1 digit after the decimal point
#

re = /(-?\d*\.\d+)/ # regular expression for input numbers

bins = Hash.new(0)

ARGF.each_line do |line|
  if re.match(line)
    v = $1.to_f
    # round off to a value with 1 digit after the decimal point
    offset = 0.5    # for round off
    offset = -offset if v < 0.0
    v = Float(Integer(v * 10 + offset)) / 10
    bins[v] += 1 # increment the corresponding bin
  end
end

bins.sort{|a, b| a[0] <=> b[0]}.each do |key, value|
  puts "#{key} #{value}"
end
```

plotting a histogram of the standard normal distribution

```
set boxwidth 0.1
set xlabel "x"
set ylabel "f(x)"
plot "box-muller-hist.txt" using 1:($2/1000) with boxes notitle, \
     1/sqrt(2*pi)*exp(-x**2/2) notitle with lines linetype 3
```

note: probability density function (PDF) of standard normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

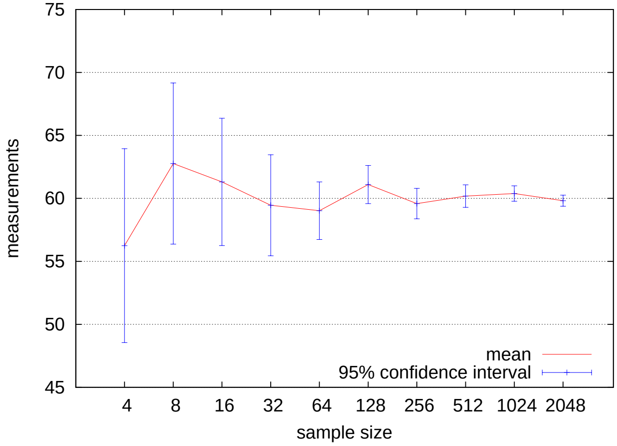
to plot a histogram

```
$ ruby box-muller.rb 10000 > box-muller-data.txt
$ ruby box-muller-hist.rb box-muller-data.txt > box-muller-hist.txt
```

then, use “box-muller-hist.plt” for plotting

the confidence interval of sample mean and sample size

the confidence interval becomes narrower as the sample size increases



the confidence interval of sample mean and sample size

plotting the confidence intervals

to make data

```
$ ruby box-muller.rb 4 60 10 | ruby conf-interval.rb > conf-interval.txt
$ ruby box-muller.rb 8 60 10 | ruby conf-interval.rb >> conf-interval.txt
$ ruby box-muller.rb 16 60 10 | ruby conf-interval.rb >> conf-interval.txt
...
$ ruby box-muller.rb 2048 60 10 | ruby conf-interval.rb >> conf-interval.txt
```

then, use “conf-interval.plt” for plotting

computing confidence intervals

```
# regular expression to read data
re = /^(\d+(\.\d+)?)/

z95 = 1.960 # z_{1-0.05/2}
z90 = 1.645 # z_{1-0.10/2}

sum = 0.0 # sum of data
n = 0 # the number of data
sqsum = 0.0 # su of squares
ARGF.each_line do |line|
    if re.match(line)
      v = $1.to_f
      sum += v
      sqsum += v**2
      n += 1
    end
end

mean = sum / n # mean
var = sqsum / n - mean**2 # variance
stddev = Math.sqrt(var) # standard deviation
se = stddev / Math.sqrt(n) # standard error
ival95 = z95 * se # intarval/2 for 95% confidence level
ival90 = z90 * se # intarval/2 for 90% confidence level

# print n mean stddev ival95 ival90
printf "%d %.2f %.2f %.2f %.2f\n", n, mean, stddev, ival95, ival90
```

plotting confidence intervals

```
set logscale x
set xrange [2:4192]
set key bottom
set xtics (4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048)

set grid ytics
set xlabel "sample size"
set ylabel "measurements"

plot "conf-interval.txt" title "mean" with lines, \
    "conf-interval.txt" using 1:2:4 title "95% confidence interval" with yerrorbars lt 3
```

assignment 1: the finish time distribution of a marathon

- purpose: investigate the distribution of a real-world data set
- data: the finish time records from honolulu marathon 2013
  - http://www.pseresults.com/events/568/results
  - the number of finishers: 22,089
- items to submit
  1. mean, standard deviation and median of the total finishers, male finishers, and female finishers
  2. the distributions of finish time for each group (total, men, and women)
     - plot 3 histograms for 3 groups
     - use 10 minutes for the bin size
     - use the same scale for the axes to compare the 3 plots
  3. CDF plot of the finish time distributions of the 3 groups
     - plot 3 groups in a single graph
  4. discuss differences in finish time between male and female. what can you observe from the data?
  5. optional
     - other analysis of your choice (e.g., discussion on differences among age groups)
- submission format: a single PDF file including item 1-5
- submission method: upload the PDF file through SFC-SFS
- submission due: 2014-11-12

honolulu marathon data set

data format

```
Place Num Chip      Lname       Fname       Country Division Div Div Sex Sex     10Km     21Km     30Km     40Km
          Time                                               Plc Tot Plc Total
-----------------------------------------------------------------------------------------------------------------
1   6     2:18:47  Chepkwony   Gilbert       KEN   MElite   1   8    1  11789 0:34:24 1:11:42 1:40:41 2:12:14
2   2     2:19:22  Chelimo     Nicholas      KEN   MElite   2   8    2  11789 0:34:25 1:11:43 1:40:41 2:12:40
3   7     2:19:38  Bushendich  Solomon       KEN   MElite   3   8    3  11789 0:34:25 1:11:43 1:40:41 2:12:51
4   4     2:20:09  Adihana     Gebretsadik   ETH   MElite   4   8    4  11789 0:34:24 1:11:42 1:40:41 2:13:16
5   8     2:20:25  Kimutai     Kiplimo       KEN   MElite   5   8    5  11789 0:34:25 1:11:42 1:40:41 2:13:21
6   1     2:21:16  Lel         Martin        KEN   MElite   6   8    6  11789 0:34:24 1:11:42 1:40:41 2:13:51
7   5     2:21:51  Tadesse     Abraham       ERI   MElite   7   8    7  11789 0:34:24 1:11:42 1:40:41 2:14:27
8   45    2:22:52  Jefferson   Fidele        USA   M35-39   1   1315 8  11789 0:34:24 1:11:43 1:40:49 2:15:29
9   25742 2:23:20  Tsukamoto   Shuji         JPN   M30-34   1   1279 9  11789 0:34:22 1:11:40 1:40:52 2:15:52
10  25767 2:31:13  Hino        Yuya          JPN   M20-24   1   702  10 11789 0:34:22 1:12:25 1:45:10 2:22:57
...
```

- Chip Time: finish time
- Category: MElite, WElite, M15-19, M20-24, ..., W15-29, W20-24, ...
  - note some runners have "No Age" for Category
- Country: 3-letter country code: e.g., JPN, USA
- check the number of the total finishers when you extract the finishers

summary

Class 4 Distribution and confidence intervals

- Normal distribution
- Confidence intervals and statistical tests
- Distribution generation
- exercise: confidence intervals
- **assignment 1**

next class

Class 5 Diversity and complexity (11/3)

- Long tail
- Web access and content distribution
- Power-law and complex systems
- exercise: power-law analysis